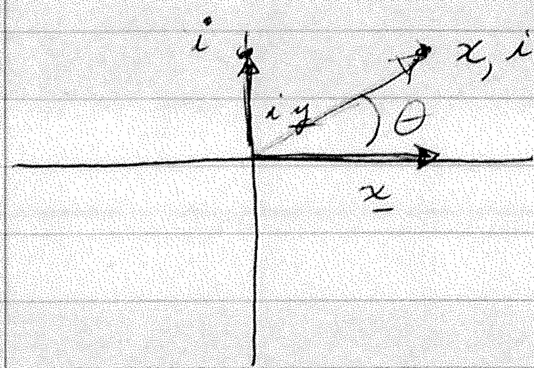
let $i = \sqrt{-1}$ an imaginary number



$$z = x + iy$$

$$|z| = \cancel{z}^{*}\,\cancel{z} = \sqrt{(x-iy)(x+iy)}$$

$$|z| = \sqrt{x^2 - ixy + ixy - i^2y^2}$$

define $|z| = r = \sqrt{x^2+y^2}$

Although i is imaginary, The coefficient in front of it (y) can have a significance in the real world.

Note $\tan\theta = y/x$ or $\theta = \tan^{-1} y/x$

Turns out That exponents of complex numbers have sine- or cosine-like behavior.

$$e^{x} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \dots$$

so $e^{ix} = 1 + ix + \frac{(ix)^2}{2} + \frac{(ix)^3}{3!} + \dots = 1 + ix - \frac{x^2}{2} - \frac{ix^3}{3!} + \dots$

looks meaningless, eh?

But let's recast our complex data point as $r(\cos\theta + i\sin\theta)$

but $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

So, it will turn out that $e^{i\theta} = \cos\theta + i\sin\theta$

Also, you can show that $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

Anyway, if someone asks you to plot $e^{i\theta}$, you can just take the real part or imaginary part & call it a cosine or sine function.

$$Re(e^{i\theta}) = \cos\theta$$
$$Im(e^{i\theta}) = \sin\theta$$

Why is this useful to us?

Because we know that the force on a viscoelastic object is the sum of two parts — one in-phase and the other out-of-phase.

In phase - $\sigma \propto \gamma$
Out-of-phase by 90° - $\sigma \propto \dot{\gamma}$

So elastic can be the real part & visco can be the part associated with the ~~[illegible]~~ imaginary term.

~~Actually, that's probably backwards.~~

define $G = G' + i G'' = |G|(\cos\delta + i \sin\delta)$

$$\delta = \tan^{-1}\left(\frac{G''}{G'}\right) \qquad \begin{aligned} G'' &= |G| \sin\delta \\ G' &= |G| \cos\delta \end{aligned}$$

$\delta = 0 \Rightarrow G' >>> G''$

$\delta = 45 \Rightarrow G' = G''$

$\delta = 90 \Rightarrow G'' >>> G'$