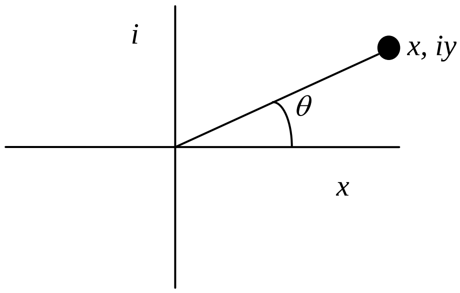
Complex Numbers Review

Let $i = \sqrt{-1}$ an imaginary number



Although *i* is imaginary, the coefficient in front of it (*y*) can have a significance on the real world.*

Note tan $\Theta = \frac{y}{x}$ or $\Theta = \tan^{-1}\frac{y}{x}$

Turns out exponents of complex numbers have sine or cosine like behavior.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

So

$$e^{ix} = 1 + ix - \frac{x^2}{2!} - \frac{ix^3}{3!} + \cdots$$

Looks meaningless, eh?

But let's recast our data point as $r \cdot (cos\Theta + isin\Theta)$

But $\quad \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \ldots$

And $\quad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \ldots$

So it will turn out that $e^{i\Theta} = cos\Theta = isin\Theta$

Also, you can show that

$$sin\Theta = \frac{e^{i\Theta} - e^{-i\Theta}}{2i}$$

$$cos\Theta = \frac{e^{i\Theta} + e^{-i\Theta}}{2}$$

Anyway, if someone asks you to plot $e^{i\Theta}$, you can just take the real part and the imaginary part and call it a cosine or sine function

$$Re(e^{i\theta}) = cos\theta$$

$$\mathrm{Im}(e^{i\theta}) = sin\theta$$

Why is this useful to us in polymer science?

Because we know that the force on a viscoelastic object is the sum of two parts: one in-phase and the other out-of-phase.

$$\text{In-phase} = \sigma \propto \gamma$$

$$\text{Out-of-phase (by 90˚)} = \sigma \propto \gamma°$$

So elastic can be the real part and visco can be the part associated with the imaginary term.

$$\text{Define } G = G' = iG" = |G|(cos\delta + isin\delta)$$

$$\delta = \tan^{-1}\frac{G"}{G'}$$

$$sin\delta = G"$$

$$cos\delta = G'$$

Thus:

$$\delta = 0 \rightarrow G' > G"$$

$$\delta = 45 \rightarrow G" = G'$$

$$\delta = 90\ G" > G'$$

*According to Feynman, who does a bang-up job of imaginary numbers, one reason we call it an imaginary plane is tied to harmonic motion. As something like a hockey puck moves in harmonic motion along the x-axis, we can liken that to rotation of our black point in the diagram. The black point and its rotation are imaginary, but the hockey puck tied to a horizontal spring is not. This real motion is the projection of the imaginary rotary motion onto the horizontal plane.